Quadrangle constructions

Task: In Geogebra software construct in the given half plane quadrangles and discuss the number of solutions in connection to the positive real parameter t.

Exercise 1: Parallelogram ABCD: $\mathrm{a} = 10\ \mathrm{cm}, |\sphericalangle \mathrm{BAC}| = 45°, |\mathrm{BD}| = \mathrm{t}\ \mathrm{cm},$

a) Solve for $t = 8$.
b) Solve with the positive real parameter t and hold a discussion.

Exercise 2 – for advanced students:

Trapezium ABC: $a = 8\ \mathrm{cm},\ \mathrm{v} = 6\ \mathrm{cm}, |\mathrm{AC}| = 7\ \mathrm{cm}, |\mathrm{BD}| = t\ \mathrm{cm}$

a) Solve for $t = 8$.
b) Solve with the positive real parameter t and hold a discussion.

Procedure:

1. Copy the task into your school exercise book. Make a rough draft, write down the procedure of the construction for the target parameter t, construct and write the number of solutions in the given half plane.

2. In Geogebra software construct the solution of the task with the circle *k* defined by the centre B and the point (with the variable radius). Choose the radius of the circle *k* so that the circle has two intersections with the straight line - as in exercise a).

3. V Geogebra software change the size of the circle radius and count the number of solutions and the individual shapes (acute-angled, obtuse-angled, right-angled triangle).

4. Write down into your school exercise book your observation in connection to the positive real parameter t, which shows the size of the radius circle *k*.

Methodological notes to solve the worksheet:

- you can add your rough drafts to solve the construction exercises on the board or assign the exercise for students in pair work
- accompany the work in Geogebra software with the collective construction on the board or on the interactive whiteboard
- discuss together the number of solutions in connection to the size of the parameter t

SOLUTION:

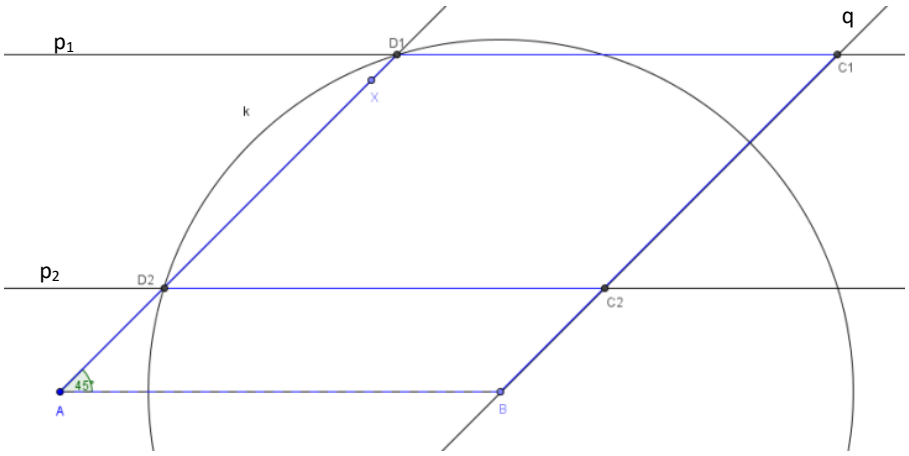
Exercise 1: Parallelogram ABCD: $\mathrm{a} = 10\ \mathrm{cm}, |\sphericalangle \mathrm{BAC}| = 45°, |\mathrm{BD}| = \mathrm{t\ cm}$

c) Solve for $t = 8$.

d) Solve with the positive real parameter t and hold a discussion.

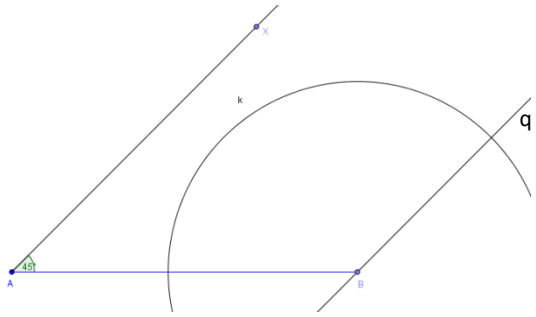
a) Construction notes:

- $AB;\ |AB| = 10\ cm$
- $\sphericalangle \mathrm{BAX};\ |\sphericalangle \mathrm{BAX}| = 45°$
- $k; k(B; 8\ cm)$
- $\mathrm{D}; \mathrm{D} \in \mathrm{k} \cap \rightarrow \mathrm{AX}$
- $p;\ p \parallel AB \wedge D \in p$
- $q;\ q \parallel AD \wedge B \in q$
- $C; C \in p \cap q$
- parallelogram $ABCD$

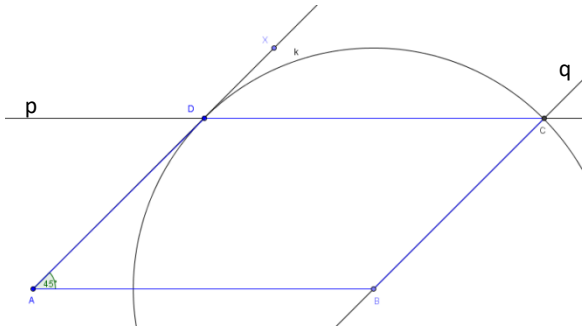


... two solutions parallelogram ABC_1D_1,parallelogram ABC_2D_2

b) Discussion (number of solutions in the given half plane):



- $t \in (0; 5\sqrt{2}) \Longrightarrow 0\ solution$



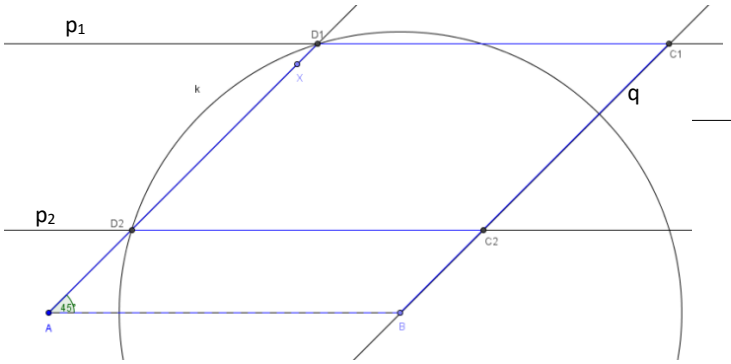
- $t \in \{5\sqrt{2}\} \Longrightarrow 1\ solution$

Question for students:

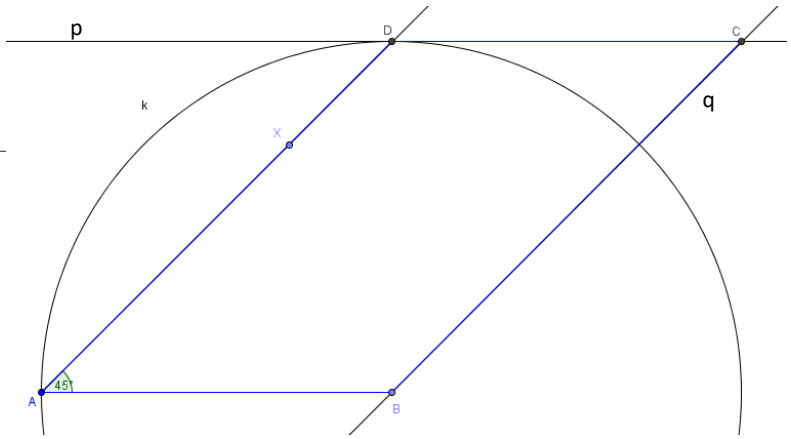
For what parameter t will this exercise have one solution?

Answer:

This exercise has one solution for parameter $t = 5\sqrt{2}$, because $|AD| = |BD| = |BC| = 5\sqrt{2}\,cm$, and for all parameters $t \geq 10$ (see below).



- $t \in (5\sqrt{2}; 10) \Longrightarrow 2\ solutions$



- $t \in \langle 10; \infty) \Longrightarrow 1\ solution$

SOLUTION:

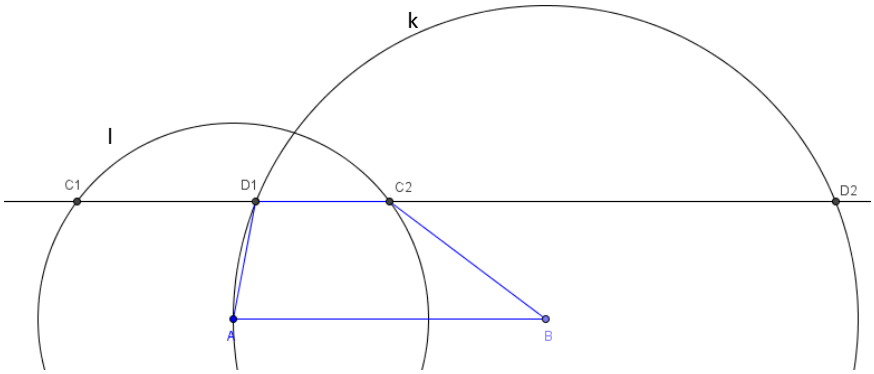
Exercise 2: Trapezium ABC: $a = 8$ cm, $v = 3$ cm, $|AC| = 5$ cm, $|BD| = t$ cm

a) Solve for $t = 8$.

b) Solve with the positive real parameter t and hold a discussion.

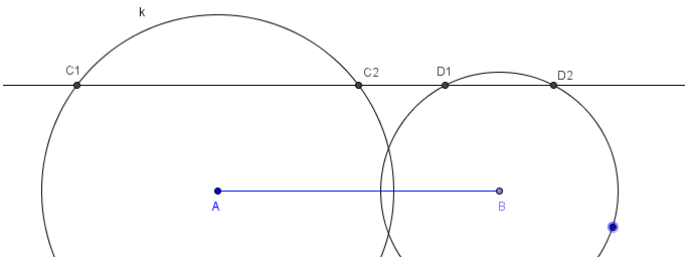
a) Construction notes:

- $AB;\ |AB| = 8\ cm$
- $p;\ p \parallel AB \wedge |p; AB| = 3\ cm$
- $l; l(A; 5\ cm)$
- $C; C \in p \cap l$
- $k; k(B; 8\ cm)$
- $D; D \in p \cap k$
- trapezium $ABCD$



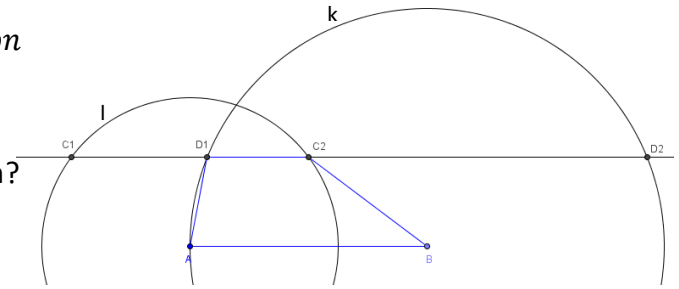
... one solution trapezium ABC_2D_1

b) Discussion (number of solutions in the given half plane):



- $t \in (0;5\rangle \implies 0\ solution$

- $t \in (5;x) \wedge x = |BC_1| = |BD_1| \implies 1\ solution$

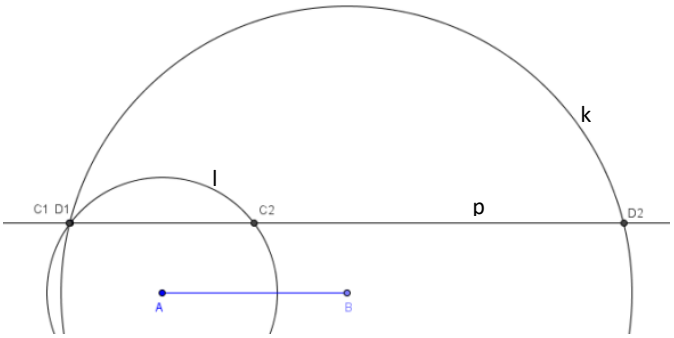


Question for students

Is it possible that ABC_2D_1 will be parallelogram and therefore this exercise won´t have any solution?

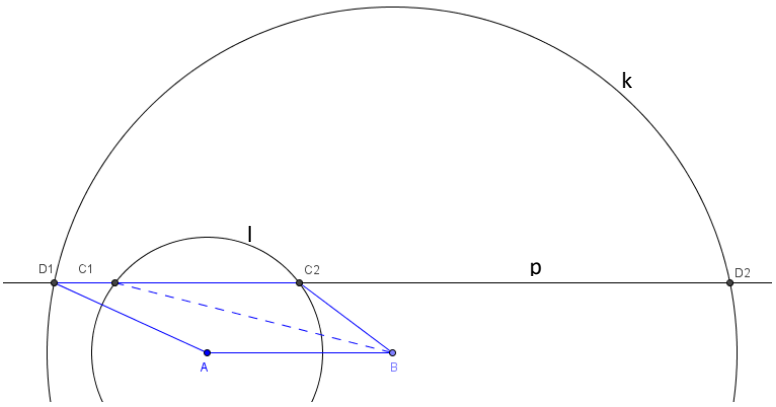
Answer:

NO, ABC_2D_1 is a parallelogram only, when $D_1 = C_1$, tzn. $|BC_1| = |BD_1| = x$ (see the next point).



- $t \in \{x\} \implies 0\ solution$
  (because $|BC_1| = |BD_1|$)

- $t \in (x;\ \infty) \implies 2\ solutions$



Question for students:

Is it possible, that ABC_2D_1 or ABC_1D_1 will be a parallelogram and therefore will the exercise have just one solution?

Answer:

YES, quadrangle ABC_1D_1 will be the parallelogram in case, when $|BC_1| = |AD_1|$.